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ABSTRACT

Contrasts or comparisons can be used to investigate specific differences between means. Contrasts, as explained by B. Thompson (1985, 1994) are coding vectors that mathematically express hypotheses. The most basic categories of contrasts are planned and unplanned. The purpose of this paper is to explain the relative advantages of using planned contrasts rather than unplanned contrasts and to illustrate several different planned contrasts that are available. Planned contrasts are designed to test predetermined specific hypotheses and have more statistical power against Type II errors than unplanned comparisons. The most common distinction among planned contrasts is whether they are orthogonal (uncorrelated) or nonorthogonal. A hypothetical data set illustrates the difference between the two. Descriptions are also given of more specific coding methods for contrasts. These methods include trend versus nontrend analyses, dummy coding, and effect coding. The use of planned contrasts is often a good alternative to traditional analysis of variance. (Contains 4 tables and 18 references.) (SLD)

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Planned Contrasts: An Overview of Comparison Methods

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Since Fisher (1925) first introduced Analysis of Variance (ANOVA) methods, they have predominated in behavioral science research. However, due to an increase in the understanding of the limitations of ANOVA and of the possibilities of more comprehensive methods, such use has declined in the past few decades (cf. Elmore & Woehlke, 1988; Goodwin & Goodwin, 1985; Willson, 1980). Nevertheless, while ANOVA methods remain a popular method of analysis, it is critical to understand what a statistically significant F gives us and what is left to discover.

The most basic question remaining when a researcher obtains a statistically significant F is where the difference lies. If only two levels have been used within a way or factor, this distinction is obvious. Beyond two levels, however, a statistically significant F does not reveal where the difference lies, and further comparisons of the means must be made to determine exactly which differences in means are statistically significant (Moore, 1983). Such contrasts are post hoc (i.e., following a statistically significant result for an omnibus test).

Contrasts or comparisons can be used to investigate specific differences between means. Contrasts, as explained by Thompson (1985, 1994), are coding vectors that mathematically express hypotheses. They may be applied to cell means or individual data. The choice of which comparison to pursue can be as creative as the researcher utilizing the contrast. The most basic categories of contrasts are planned and unplanned contrasts. It is the purpose of this paper to explain the relative advantages of using planned contrasts rather than unplanned contrasts and to illustrate several different planned contrasts that are available.

Planned versus Unplanned Contrasts

Unplanned contrasts, which are also referred to as posteriori, unfocused, or post hoc contrasts, have been described rather creatively by textbook authors as "data snooping"(Kirk,

1968, p. 73), a "fishing expedition" (Minium & Clarke, 1982, p. 321), "milking a set of results" (Keppel, 1982, p. 150), and "data 'sifting'" (Keppel, 1982, p. 93), among other things. Such descriptions do not speak highly of the method. It can be argued that when one does not know what he's looking for, unplanned contrasts are helpful in making an unanticipated but important discovery (Keppel, 1973). It is hoped, however, that very few occasions arise in which the investigator has no reasonable specific expectations prior to the experiment regarding where differences will arise.

Planned contrasts, on the other hand, are designed to test predetermined specific hypotheses. Aside from making intuitive sense, planned contrasts, a priori contrasts, or focused contrasts have two additional advantages over unplanned comparisons (Thompson, 1994). First of all, planned comparisons have more statistical power against Type II errors than unplanned comparisons. Unplanned contrasts take into consideration many or all possible contrasts. As a result, they must correct for all contrasts made, including ones in which the researcher is not really interested. Planned comparisons, however, are only required to account for the comparisons of interest, thereby increasing power against Type II errors. Secondly, as Thompson (1988, 1994) points out, "planned comparisons tend to force the researcher to be more thoughtful in conducting research" (1988, p. 100). He explains that the researcher must hypothesize beforehand which comparisons are most important, given a limited number of comparisons allowed. Ultimately, of course, it is the researcher's decision which comparison is best suited to answer the hypotheses being explored.

The remainder of the present paper examines various methods of performing planned contrasts. The broad categories of orthogonal and nonorthogonal comparisons are explained first,

followed by descriptions of more specific coding methods. These include trend versus non-trend analyses, dummy coding, and effect coding.

Orthogonal and Nonorthogonal Contrasts

As stated previously, planned contrasts can take many forms. The most common distinction among planned comparisons is whether they are orthogonal or nonorthogonal. Comparisons are said to be orthogonal, or uncorrelated, "if knowledge of the outcome of one contrast in no way helps to predict the outcome of the second contrast" (Hinkle, Wiersma, & Jurs, 1998, p. 401). Contrasts are determined to be orthogonal if the cross-products of the contrast coefficients used to test the contrast hypothesis sum to zero and also when the coefficients of each contrast considered separately also sum to zero (Howell, 1992; Thompson, 1994).

Consider the following example from the study reported by Carr and Thompson (1996). The study involved three equal-sized groups of 15 students who completed a reading test as the dependent measure. Two a priori (also called planned or focused) contrasts were used:

	C1	C2	C1xC2
8th grade regular education students	0	+2	0
8th grade learning disabled (LD) students	-1	-1	+1
5th grade regular education students	+1	-1	-1
Mean	0	0	
Sum	0	0	0

The first contrast ("C1") tested whether the mean of the 15 8th grade LD students on the reading test differed from the mean of the 15 5th grade regular education students. The second contrast ("C2") tested whether the mean of the 15 8th grade regular education students differed from the reading test mean of the 30 students from the other two groups combined.

Given a developmental delay view of learning disability, the researchers expected to not reject and to achieve a small effect size for the hypothesis tested by the C1 contrast. However, the

researcher expected to achieve both statistical significance and a large effect size for the C2 contrast. Both expectations were confirmed.

It can readily be demonstrated that the two contrasts were orthogonal (uncorrelated). One formula for r is:

$$r_{xy} = [(\Sigma(X_i - \bar{X})(Y_i - \bar{Y})) / n - 1] / SD_x * SD_y)$$

As long as neither SD is zero, if the numerator of the numerator in this formula (i.e., $[\Sigma (X_i - \bar{X})(Y_i - \bar{Y})]$) equals zero, r will be zero. Contrasts are always constructed so that the means of the contrast variables are zero. Thus, if the sum of the cross-products of the contrast variables (i.e., $[\Sigma (X_i)(Y_i)]$) equals zero, r equals zero. As noted above, these conditions were met for these contrasts.

With orthogonal contrasts, the number of contrasts in the design is limited to the number of levels in a given way minus 1. For example, with a 4x3 design, k=4 levels for the "A" way, so 4-1, or 3 orthogonal contrasts are possible for this way (but 2 are possible for the "B" way). Again, all contrasts are between two means. It should also be noted, remembering all hypotheses are uncorrelated or independent of each other and remembering that k-1 contrasts are permitted, that the sum of squares for each of the comparisons for a given way sum to equal exactly the sum of squares for that way (Hinkle et al., 1998)

The sections that follow describe special cases for coding contrasts. Some contrasts, such as trend analysis, are orthogonal. Contrasts can also be nonorthogonal, however. As these comparisons are discussed, they will be illustrated by using a common data set. The purpose of the present illustration is to highlight the similarities and differences across types of comparisons. The data used will be a hypothetical set of data developed by Tucker (1991). The study involved a six-level one-way design with two participant per group, as seen in Table 1. From this design, we

can envision what a researcher might hypothesize and how different methods of coding contrasts would answer those questions. The hypothetical data involves as a dependent variable a measure of attitudes toward school. Six groups are included within the one way: students, teacher aides, teachers, principals, superintendents, and board members.

Contrast Coding

Contrast coding addresses particular questions for the researcher. Contrast coding can be either orthogonal or nonorthogonal. Using orthogonal contrasts for these data, we can test k-1 or five hypotheses, though of course several alternative sets of five hypotheses are possible. For simplicity's sake, we can examine those hypotheses proposed in Tucker (1991). The null hypotheses are as follows:

C1: The mean of the 2 students' attitudes equals the mean of teacher aides' attitudes.

C2: The mean of the 4 students' and teacher aides' attitudes equals the mean of the 2 teachers' attitudes.

C3: The mean of the 6 students', teacher aides', and teachers' attitudes equals the mean of the 2 principals' attitudes.

C4: The mean of the 8 students', teacher aides', teachers', and principals' attitudes equals the mean of the 2 Superintendents' attitudes.

C5: The mean of the 10 students', teacher aides', teachers', principals', and superintendents' attitudes equals the mean of the 2 Board members' attitudes.

These are simply a few of many hypotheses that could be generated. Other examples will follow.

Trend Analysis

As the name implies, trend or polynomial contrast analysis is used to assess the trends defined by the means of the dependent variable. Hinkle et al. (1998) noted that trend analysis

> allows the researcher to answer questions such as
>
> 1. Do the means of the treatment group increase (decrease) in linear fashion with an increase in the level of the independent variable?
>
> 2. Is the trend linear or nonlinear?
>
> 3. If the trend is nonlinear, what degree equation (polynomial) is required to fit the data? (p. 406)

Coefficients for orthogonal polynomials are used to test these comparisons. Again, k-1 comparisons are allowed. However, the hypothetical data presented here is not conducive to trend analysis. First of all, the levels of the data are discrete categories of people and not equally spaced different levels of a quantitative treatment (Ferguson & Takane, 1989).

Let's say that the six levels were number of children in the family, from 1 to 6, and the dependent variable involved scores on a measure of family cohesiveness. Now the way is quantitative and the levels are equally spaced. The first polynomial or trend contrast would test whether the 6 means defined a straight line (i.e., either sloped up or down uniformly across the six levels. The second polynomial contrast would test whether the six means descended, turned the middle, and the ascended (e.g., 10, 20 , 30, 20, 10) or vice versa (e.g., 40, 30, 20, 30, 40). Each successive contrast tests a pattern in which the means "bend" more times (e.g., linear tests for no

bends, quadratic tests for one bend in the middle of the levels, cubic tests for two bends, quartic tests for three bends).

The coefficients for a three, four, and five level analysis can be seen in Table 2. Example hypotheses for trend analysis orthogonal contrasts would be:

1. Quantity of alcohol consumption and aggressiveness in men has a curvilinear relationship. Three 6oz. drinks have little effect on a measure of aggressiveness, six 6oz. drinks have a substantially higher measure of male aggressiveness, and nine 6oz. drinks are associated with a markedly reduced level of aggressiveness (a quadratic trend).
2. As practice duration increases, basketball scoring efficiency increases (a curvilinear trend).

Dummy Coding

Dummy coding is generally seen as the least complex system of coding (Hinkle & Oliver, 1986; Pedhazur, 1982). In basic terms, "in any given vector, membership in a group or category is assigned 1, while nonmembership in the category is assigned zero" (Pedhazur, 1982, p. 274). The contrasts for dummy coding may be seen in Table 3.

Notice that the final level is not depicted by a comparison. For a six-level way, only 5 (k-1) vectors, or comparisons are depicted. No information is lost, however. Knowledge of a sixth vector, with the sixth level receiving 1's and the other levels receiving 0's would not provide any information not already obtained from the first five vectors.

That is, each contrast represents one degree of freedom; when k-1 contrasts of any kind for a given way have been exhausted, the full sum-of-squares for that way will have been determined. Because only 5 comparisons are made, however, do not equate dummy coding

automatically with orthogonal contrasts. Visual inspection reveals that none of the vector coefficients sum to zero and the sums of the cross-products are not 0; therefore these contrasts are not independent. Dummy coding is used simply to test for differences between a level and the scores of the group as a whole (i.e., the grand mean across all levels).

Effect Coding

Effect coding is more complex than dummy coding. It is most helpful when a control group is used for comparison with other groups. 1's, 0's, and -1's are used to determine the treatment effects. The final group (control group, if used) receives -1's in all comparisons, while the comparison group, or level, receives 1's. The rest of the groups receive 0's. The comparisons for the hypothetical data using effect coding are illustrated in Table 4.

It should be noted that which level is determined as the control or comparison group is left to the discretion of the researcher. In the present example, the attitudes of the board members were compared with the attitudes of the other groups. Any of the six groups could serve as a reference point. For example, the researcher could have hypothesized that the attitudes of students toward school would be different than the attitudes of the adults (the other 5 groups). In this instance, students would serve as the reference group and would receive codes of -1 in all 5 comparisons.

Nonorthogonal Contrasts: Control of Experimentwise Error

As the name implies, all contrasts that do not meet the conditions for orthogonal contrasts are considered non-orthogonal contrasts. Reaction as to whether anything but orthogonal contrasts should be used as planned comparisons has been mixed (Pedhazur, 1982; Thompson, 1994). A good case for the use of nonorthogonal contrasts can be found in Thompson (1994). In

general, he and others (e.g., Huberty & Morris, 1988; Winer, 1971) who support the use of nonorthogonal contrasts emphasize the importance of asking important and creative questions of the data. It is the design of the experiment, the nature of the questions asked, and not the analysis that determines appropriateness.

The predominant unknown in nonorthogonal contrasts is experimentwise error (EW) (Pedhazur, 1982; Thompson, 1994). Nonorthogonal comparisons are not limited to k-1 hypotheses. Several possible avenues to control for EW are presented by Thompson (1994), including a broad guideline of reasonableness, restricting EW inflation to that obtained by all allowed omnibus tests, or invoking effectwise level limits. Another option mentioned is to use a Bonferroni (or Dunn) correction to control for EW inflation. The rule proposed for this approach is

> Once the multiple R between nonorthogonal contrasts for an omnibus effect and the relevant cell information (e.g., A=1, A=2) equals 1.0, and if not all contrasts for the effect have been used to predict cell assignment, then invoke a Bonferroni correction for the EW inflation (Huberty, 1987). (Thompson, 1994, p. 18)

The Bonferroni t adjustment corrects comparisons so that EW does not exceed the sum of separate comparison probabilities (Howell, 1992). For perfectly uncorrelated hypotheses (such as orthogonal contrasts or the omnibus tests in a balanced classical ANOVA design) or dependent variables, the experimentwise error rate (α_{EW}) can be calculated using the Bonferroni inequality:

$$\alpha_{EW} = (1 - \alpha_{TW})^K,$$

where k is the number of perfectly uncorrelated hypotheses being tested at a given testwise alpha level (α_{TW}). For example, if five uncorrelated/orthogonal contrasts (or omnibus effects in a balanced ANOVA design) are tested using data from a single sample, each at the $\alpha_{TW} = .05$ level of statistical significance, the experimentwise Type I error rate will be:

$$\begin{aligned}\alpha_{EW} &= 1 - (1 - \alpha_{TW})^K \\ &= 1 - (1 - .05)^5 \\ &= 1 - (.95)^5 \\ &= 1 - (.95)(.95)(.95)(.95)(.95)) \\ &= 1 - .773780 \\ \alpha_{EW} &= .226219\end{aligned}$$

Note that 22.6% is approximately equal to k=5 times $\alpha_{TW} = .05$ (i.e., 25%). This suggests the so-called Bonferroni correction by which we lower the original α_{TW} to a new value, α_{TW}^*, such that the α_{EW} will now be lower than the original α_{TW}, as follows:

$$\alpha_{TW}^* = \alpha_{TW} / k.$$

Here α_{TW}^* will be .05/5, or .01. Now the new α_{EW} will equal:

$$\begin{aligned}\alpha_{EW} &= 1 - (1 - \alpha_{TW}^*)^K \\ &= 1 - (1 - .01)^5 \\ &= 1 - (.99)^5 \\ &= 1 - (.99)(.99)(.99)(.99)(.99)) \\ &= 1 - .950990 \\ \alpha_{EW} &= .049009,\end{aligned}$$

which is less than the original $\alpha_{TW} = .05$. For more detail, see Thompson (1994).

Summary

When considering planned contrasts, the researcher's greatest limitation is one's own creativity and ingenuity in developing hypotheses. This paper highlighted several issues regarding planned contrasts, specifically as against unplanned contrasts. Several methods of coding planned contrasts were discussed in relation to a hypothetical set of data. The merits of orthogonal and

nonorthogonal contrasts were also presented briefly, including the need for a correction for EW error by using Bonferroni's correction for some nonorthogonal comparisons. It is hoped that this information will enlighten and inspire the reader to investigate further, alternative methods to traditional analysis of variance methods.

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Table 1. Hypothetical Data for Attitudes toward School Study (n=12)
Contrast Coding

				Contrasts				
Group	Level	ID	DV	C1	C2	C3	C4	C5
Students	1	1	15	-1	-1	-1	-1	-1
		2	25	-1	-1	-1	-1	-1
Teacher Aides	2	3	15	1	-1	-1	-1	-1
		4	25	1	-1	-1	-1	-1
Teachers	3	5	15	0	2	-1	-1	-1
		6	25	0	2	-1	-1	-1
Principals	4	7	15	0	0	3	-1	-1
		8	25	0	0	3	-1	-1
Superintendents	5	9	15	0	0	0	4	-1
		10	25	0	0	0	4	-1
Board Members	6	11	30	0	0	0	0	5
		12	40	0	0	0	0	5

(Tucker, 1991)

Table 2. Coefficients for Orthogonal Polynomials

	k=3				
Linear	-1	0	1		
Quadratic	1	-2	1		
	k=4				
Linear	-3	-1	1	3	
Quadratic	1	-1	-1	1	
Cubic	-1	3	-3	1	
	k=5				
Linear	-2	-1	0	1	2
Quadratic	2	-1	-2	-1	2
Cubic	-1	2	0	-2	1
Quartic	1	-4	6	-4	1

Table 3. Hypothetical Data for Attitudes toward School Study (n=12)
Dummy Coding

				Contrasts				
Group	Level	ID	DV	C1	C2	C3	C4	C5
Students	1	1	15	1	0	0	0	0
		2	25	1	0	0	0	0
Teacher Aides	2	3	15	0	1	0	0	0
		4	25	0	1	0	0	0
Teachers	3	5	15	0	0	1	0	0
		6	25	0	0	1	0	0
Principals	4	7	15	0	0	0	1	0
		8	25	0	0	0	1	0
Superintendents	5	9	15	0	0	0	0	1
		10	25	0	0	0	0	1
Board Members	6	11	30	0	0	0	0	0
		12	40	0	0	0	0	0

Table 4. Hypothetical Data for Attitudes toward School Study (n=12)
Effect Coding

				Contrasts				
Group	Level	ID	DV	C1	C2	C3	C4	C5
Students	1	1	15	1	0	0	0	0
		2	25	1	0	0	0	0
Teacher Aides	2	3	15	0	1	0	0	0
		4	25	0	1	0	0	0
Teachers	3	5	15	0	0	1	0	0
		6	25	0	0	1	0	0
Principals	4	7	15	0	0	0	1	0
		8	25	0	0	0	1	0
Superintendents	5	9	15	0	0	0	0	1
		10	25	0	0	0	0	1
Board Members	6	11	30	-1	-1	-1	-1	-1
		12	40	-1	-1	-1	-1	-1